# NAG Toolbox for MATLAB

# g13aa

## 1 Purpose

g13aa carries out non-seasonal and seasonal differencing on a time series. Information which allows the original series to be reconstituted from the differenced series is also produced. This information is required in time series forecasting.

## 2 Syntax

```
[xd, nxd, ifail] = g13aa(x, nd, nds, ns, 'nx', nx)
```

## 3 Description

Let $\nabla^d \nabla_s^D x_i$ be the $i$th value of a time series $x_i$, for $i = 1, 2, \ldots, n$ after non-seasonal differencing of order $d$ and seasonal differencing of order $D$ (with period or seasonality $s$). In general,

$$\begin{aligned} \nabla^d \nabla_s^D x_i &= \nabla^{d-1} \nabla_s^D x_{i+1} - \nabla^{d-1} \nabla_s^D x_i && d > 0 \\ \nabla^d \nabla_s^D x_i &= \nabla^d \nabla_s^{D-1} x_{i+s} - \nabla^d \nabla_s^{D-1} x_i && D > 0 \end{aligned}$$

Non-seasonal differencing up to the required order $d$ is obtained using

$$\begin{aligned} \nabla^1 x_i &= x_{i+1} - x_i && \text{for } i = 1, 2, \ldots, (n-1) \\ \nabla^2 x_i &= \nabla^1 x_{i+1} - \nabla^1 x_i && \text{for } i = 1, 2, \ldots, (n-2) \\ &\vdots \\ \nabla^d x_i &= \nabla^{d-1} x_{i+1} - \nabla^{d-1} x_i && \text{for } i = 1, 2, \ldots, (n-d) \end{aligned}$$

Seasonal differencing up to the required order $D$ is then obtained using

$$\begin{aligned} \nabla^d \nabla_s^1 x_i &= \nabla^d x_{i+s} - \nabla^d x_i && \text{for } i = 1, 2, \ldots, (n-d-s) \\ \nabla^d \nabla_s^2 x_i &= \nabla^d \nabla_s^1 x_{i+s} - \nabla^d \nabla_s^1 x_i && \text{for } i = 1, 2, \ldots, (n-d-2s) \\ &\vdots \\ \nabla^d \nabla_s^D x_i &= \nabla^d \nabla_s^{D+1} x_{i+s} - \nabla^d \nabla_s^{D+1} x_i && \text{for } i = 1, 2, \ldots, (n-d-D \times s) \end{aligned}$$

Mathematically, the sequence in which the differencing operations are performed does not affect the final resulting series of $m = n - d - D \times s$ values.

## 4 References

None.

## 5 Parameters

### 5.1 Compulsory Input Parameters

1: **x(nx) – double array**

The undifferenced time series, $x_i$, for $i = 1, 2, \ldots, n$.

2: **nd – int32 scalar**

$d$, the order of non-seasonal differencing.

*Constraint*: **nd** $\geq 0$.

3: **nds – int32 scalar**

*D*, the order of seasonal differencing.

*Constraint*: **nds** $\geq 0$.

4: **ns – int32 scalar**

*s*, the seasonality.

*Constraints*:

if **nds** $> 0$, **ns** $> 0$;
if **nds** $= 0$, **ns** $\geq 0$.

### 5.2 Optional Input Parameters

1: **nx – int32 scalar**

*Default*: The dimension of the arrays **x**, **xd**. (An error is raised if these dimensions are not equal.)

*n*, the number of values in the undifferenced time series.

*Constraint*: **nx** $>$ **nd** $+$ (**nds** $\times$ **ns**).

### 5.3 Input Parameters Omitted from the MATLAB Interface

None.

### 5.4 Output Parameters

1: **xd**(**nx**) **– double array**

The differenced values in elements 1 to **nxd**, and reconstitution data in the remainder of the array.

2: **nxd – int32 scalar**

The number of differenced values in the array **xd**.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

## 6 Error Indicators and Warnings

Errors or warnings detected by the function:

**ifail** $= 1$

On entry, **nd** $< 0$,
or **nds** $< 0$,
or **ns** $< 0$,
or **ns** $= 0$ when **nds** $> 0$.

**ifail** $= 2$

On entry, **nx** $\leq$ **nd** $+$ (**nds** $\times$ **ns**).

## 7 Accuracy

The computations are believed to be stable.

## 8 Further Comments

The time taken by g13aa is approximately proportional to $(\mathbf{nd} + \mathbf{nds}) \times \mathbf{nx}$.

## 9 Example

```
x = [120;
     108;
     98;
     118;
     135;
     131;
     118;
     125;
     121;
     100;
     82;
     82;
     89;
     88;
     86;
     96;
     108;
     110;
     99;
     105];
nd = int32(2);
nds = int32(1);
ns = int32(4);
[xd, nxd, ifail] = g13aa(x, nd, nds, ns)
```

```
xd =
   -11
   -10
    -8
     4
    12
    -2
    18
     9
    -4
    -6
    -5
    -2
   -12
     5
     2
   -10
   -13
    17
     6
   105
nxd =
          14
ifail =
           0
```